Comparison of stochastic identification methods applied to the natural response of Millau Viaduct

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ABSTRACT: The efficiency of different alternative output-only modal identification methods applied to data collected during the dynamic tests performed at the Millau Viaduct at the commissioning stage is evaluated here. In particular, modal estimates identified on the basis of an ambient vibration test are correlated with data from free vibration tests and from modal properties obtained numerically at the design stage.

1 INTRODUCTION

The Viaduct of Millau (Figure 1) is an outstanding multi-span cable-stayed bridge inserted in the highway A75, linking Clermont-Ferrand and Béziers, which integrates a new connection between Northern Europe and East Spain.

Owing to the necessity of achieving very accurate estimates of the dynamic properties of this structure (natural frequencies, mode shapes and modal damping ratios), in order to experimentally validate the finite element modelling developed at the design stage, dynamic tests were performed at the commissioning phase, under the coordination of CSTB (Flamand and Grillaud 2005). The Laboratory of Vibrations and Monitoring (ViBest) of FEUP has been invited to collaborate, under contract, and performed in particular an independent ambient vibration test.

This paper describes some important details of the instrumentation used and the dynamic tests performed, and analyses the performance of different alternative output-only modal identification methods applied to the natural response of the Millau Viaduct, correlating the modal estimates achieved by an ambient vibration test with the estimates obtained from free vibration tests and from the finite element modelling developed at the design stage.

2 THE MILLAU VIADUCT

The Millau Viaduct is a large cable-stayed road-bridge that spans the valley of the River Tarn near Millau in southern France (Figure 1). Designed by the architect Norman Foster and the bridge engineer Michel Virlogeux, it is the tallest vehicular bridge in the world, with one pylon's top rising at 343m above Tarn level. The 8 cable-stayed spans with a total length of 2460m make this bridge also the longest cable-stayed bridge in the world. The 32m wide and 4.2m thick deck has a continuous steel box section over the 2460m of extension. The six central spans have a length of 342m, while the outer spans are 204m long. The seven concrete piers range in height from 77 to 245m, supporting 87m tall steel pylons.

Figure 1. General view Millau Viaduct.

3 DESCRIPTION OF THE DYNAMIC TESTS

3.1 *Instrumentation*

Owing to the predicted dynamic properties of this multi-span cable-stayed bridge, the type of dynamic tests required, the time available to perform them (3 days) and the number of sensors available, the CSTB team defined a Plan of Tests trying to conjugate the instrumentation of the permanent monitoring system installed by the SITES company with a complementary equipment. This plan, schemmatically represented at Figure 2, comprised 21 accelerometers disposed in 5 out of the 8 spans, measuring vertical (13 measurement sections on the deck) and longitudinal (1 measurement section on the deck and 8 in the pylons) accelerations. 12 of these accelerometers (designated by S in Figure 2) belonged to the system installed by SITES, located in the third and fourth spans supported by the highest piers, whereas the remaining 9 (indicated as C in Figure 2) belonged to CSTB and FEUP, and were installed in two consecutive spans and at the corresponding pylons, where the data acquisition system from CSTB was installed.

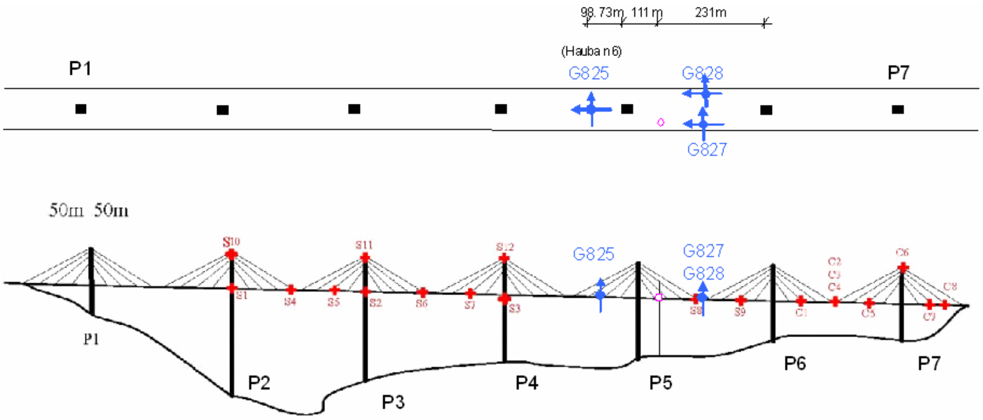

Figure 2. Location of the instrumentation used in the free vibration test

All the sensors were installed inside the box girder, along the longitudinal axis of the viaduct. Only one pair of accelerometers (C2, C4) was located at the lateral edges of the structure, with the purpose of identifying some torsional modes.
Despite the different characteristics of the data acquisition systems from SITES and CSTB, it was possible to synchronize them. A sampling frequency of 40Hz was employed.
An ambient vibration test was developed complementarily using 4 seismographs, including tri-axial accelerometers, three of them located at positions G825, G826 and G827, indicated in Figure 2, during the free vibration test. These seismographs, duly synchronized using GPS sensors, were placed outside the deck, as shown in Figure 3.

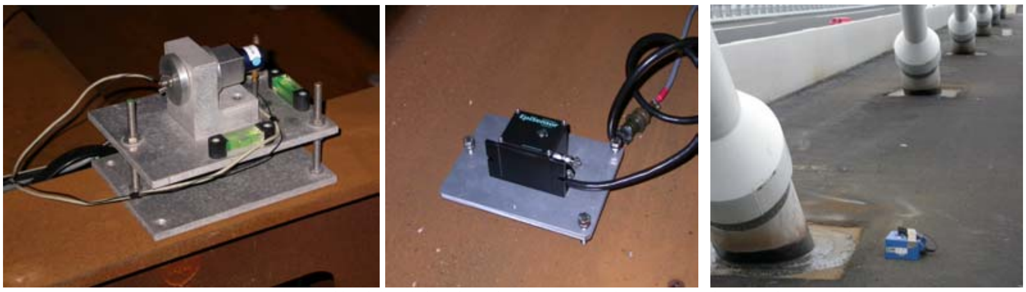

Figure 3. Accelerometers installed inside the deck and seismograph used outside

3.2 *Ambient Vibration Test*

This test was performed on the 24th November 2004, using the 4 previously mentioned seismographs. With the purpose of identifying as many modes of vibration as possible, essentially of vertical and transversal bending nature, two of those tri-axial sensors were used as references (sections R1 and R2, Figure 4), keeping them in fixed positions, while the other two were successively placed in each one of the remaining 26 sections schematically indicated in Figure 4. The seismographs were programmed using a laptop in order to acquire signals with a sampling rate of 100Hz in periods of 960s every 20 minutes, the last 4 minutes being used to change the position of the moving sensors for the following measurement setup.

The test was completed in 7h30, and during this period the wind speed was always very low and the traffic very sporadic, as the bridge had not been open to traffic yet.

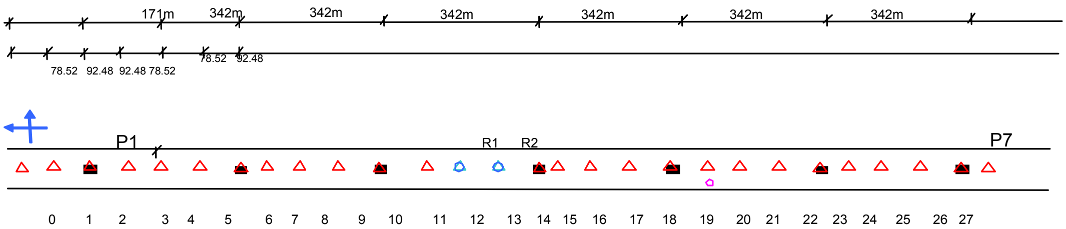

Figure 4. Instrumented sections of the deck in the ambient vibration test

3.3 *Free Vibration Tests*

Figure 5. Free vibration test: tensioned cable, cut from inside the box girder

The free vibration test was performed on the 25th November 2004, and consisted of the sudden rupture of a tensioned cable connected to the deck. This operation was conducted first with a cable tensioned at 600kN and then repeated with a tensioning of the cable to 1000kN. In both cases the wind speed was low, oscillating between 2 and 5m/s. The structural response was measured during 960s

Figure 2 indicates the measurement sections used, as well as the point of the deck chosen to hang the cable, which is located at about 1/3 of the P5-P6 span, eccentrically so as to induce symmetric and anti-symmetric bending modes of vibration, and also torsional modes in the free vibration response. In the vicinity of this point, the measured value of maximum vertical acceleration was about 1m/s^2, and the corresponding peak displacement (obtained by integration) was about 50 mm. Figure 5 shows some pictures from the test.

4 OUTPUT-ONLY IDENTIFICATION FROM THE AMBIENT VIBRATION TEST

4.1 *Preliminary frequency domain analysis*

Preliminary frequency domain analysis was developed in order to identify the most relevant natural frequencies and to analyse the variation of the frequency content of the collected time series during the 13 setups.

For each setup, two spectral matrices were calculated, one for vertical and another one for lateral accelerations. Then, a singular value decomposition of the spectral matrices was performed. Figure 6 shows the average singular values and colour maps with the variation of the 1st singular values during the ambient vibration test, for both directions. These plots show the existence of a large number of modes in the frequency range 0.1-1.0 Hz. Inspection of the colour maps also allows to notice that some modes are not clearly represented in all the setups, which helps to understand the results presented in the next section.

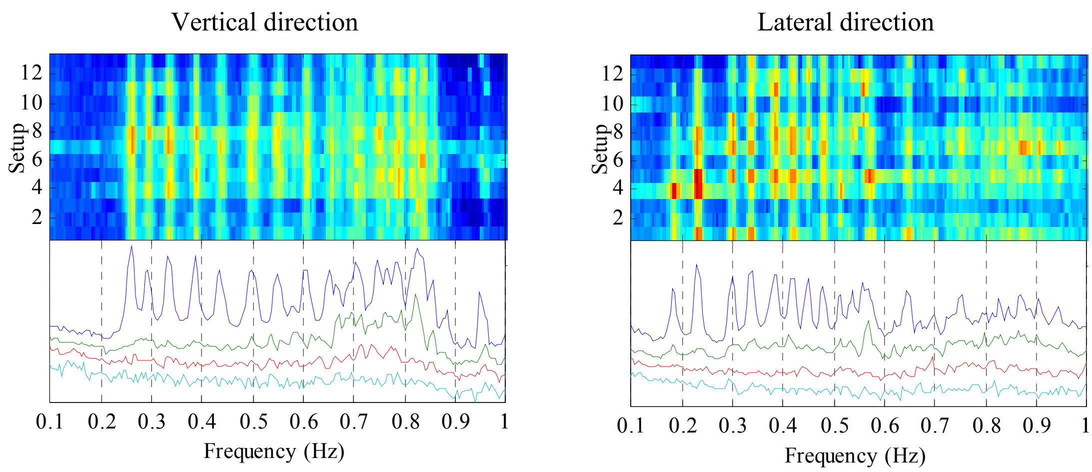

Figure 6. Average singular values of the spectra matrices and colour maps with the variation of the 1st singular values during the ambient vibration test.

4.2 *PolyMax method*

The PolyMax method was first developed to be used in the context of input-output modal identification. Recently, it was adapted for ambient vibration tests. In the used version of the method, a frequency domain parametric model is fitted to the spectra of the measured structural responses. Its algorithm is basically developed in two steps: in a first instance, an optimization problem leads to a stabilization diagram containing frequency, damping and operational reference factors; in a second step, the mode shapes are found from the resolution of a least-squares problem, based on the user selection of stable poles. A detailed description of this method can be found in reference (Peeters & Auweraer 2005).

This output-only modal identification method was applied to the data collected in the Millau viaduct using the Test.Lab software developed by LMS International (LMSInternational 2005).

The bases of the method are cross spectra between signals measured simultaneously at different locations. These were calculated from the Discrete Fourier Transform of the cross correlation functions windowed by exponential functions.

The calculated spectra are organized in spectral matrices that have *l* lines and *r* columns, *l* being the number of degrees of freedom measured in each setup and *r* the number of degrees of freedom selected for references. In the present study, 4 references were used: lateral and vertical accelerations at the two reference sections.

For each setup, the calculated spectral matrix is used to produce a stabilization diagram that allows the identification of natural frequencies and modal damping ratios. Figure 7 shows one of the 13 stabilization diagrams. In this diagram, the most relevant alignments with stable poles are selected and the identified natural frequencies and modal damping ratios are presented at the left side of the picture. Inspection of the stabilization diagram shows that this method was able to identify 20 modes in the frequency range between 0.1 and 0.8 Hz, including two pairs of modes with closely spaced natural frequencies (indicated in the picture with the red and black boxes).

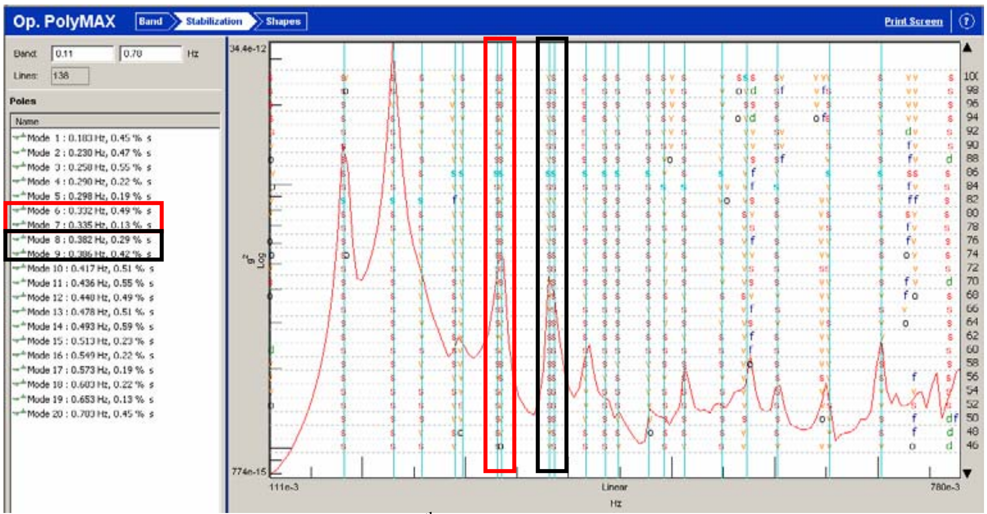

Figure 7. Stabilization diagram of the 4th setup - PolyMax method

Following this procedure, the data of each setup originates a set of modal parameters. To have the global picture of the mode shapes, a procedure was adopted to find the correspondence between the modal parameters identified in the different setups. This is not straightforward, since not all the modes are identified in all the setups and because, for modes with close natural frequencies, the frequencies can change their relative order during the setups.

In the present application, the pairing of the modal parameters identified in the 13 setups was developed considering the modal parameters identified in one representative setup (setup4) as target values. Then, in all the other setups the poles with natural frequencies in the vicinity of the target values that presented higher MAC values were chosen. The MAC coefficients were calculated using the reference degrees of freedom.

Figure 8 presents all pairs of identified natural frequencies and modal damping ratios. The final estimates of the natural frequencies and of the modal damping coefficients (values also presented at Figure 8) are the averages of the values provided by each setup that were associated using the previously described procedure.

The global configurations of the mode shapes are obtained concatenating the components identified in each setup, which are related between each other through the ordinates at the reference sensors. Figure 9 and Figure 10 show some of the identified mode shapes. In particular, the mode shapes of the pairs with closely spaced frequencies are represented to show that this identification technique was able to completely separate the modes. Some modal ordinates are missing in the representation of the 4th vertical mode shape, because this mode was not identified in one of the setups. This is one of the disadvantages of processing the setup data separately. Another disadvantage is that the application of the identification method was very time consuming, as 13 stabilization diagrams had to be analyzed.

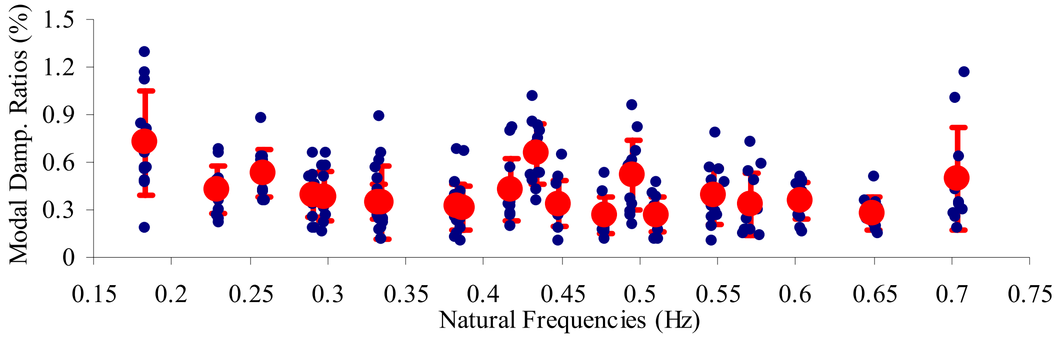

Figure 8. Identified natural frequencies and modal damping ratios: values estimated in each setup and final results represented by the average values and by an interval of variation for the modal damping ratios based on the standard deviation (average +/- standard deviation).

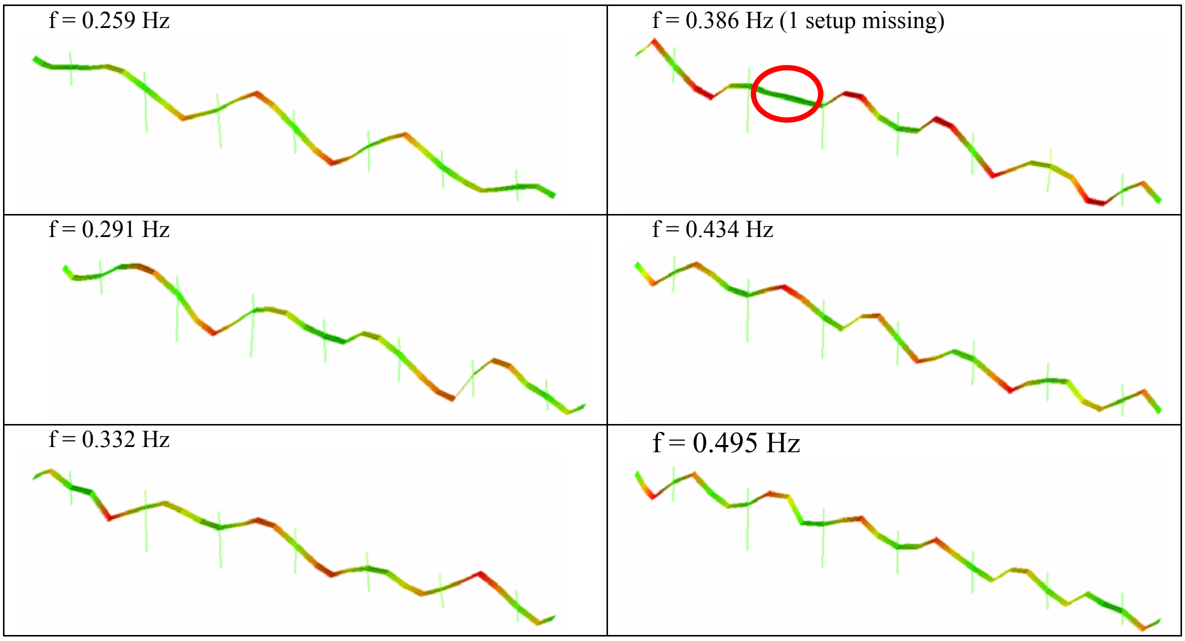

Figure 9. First six vertical identified mode shapes

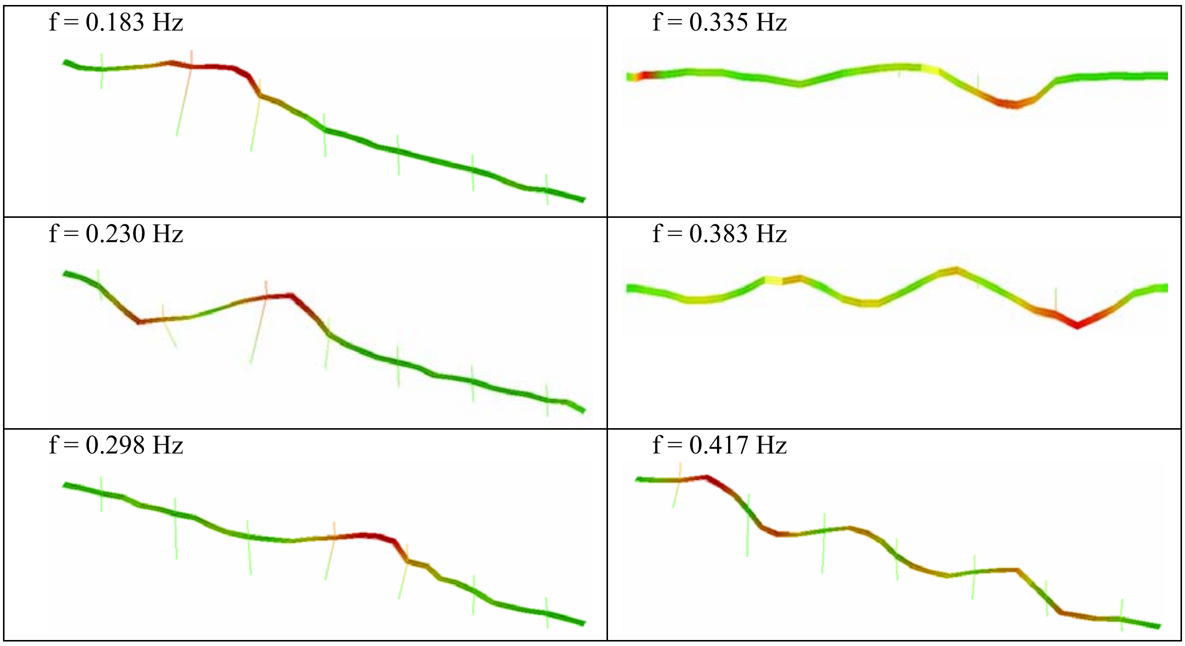

Figure 10. First six lateral identified mode shapes

To overcome the limitation of the independent analysis of each setup, it is possible to perform a single analysis using a matrix that contains the spectra matrices of all the setups. This procedure was followed with success by Peeters et al. (2006) and was previously studied by Parloo (2003). However, it is not implemented in the Lab.Test yet.

Therefore, the use of a single analysis will be presented in the next section, but using the Stochastic Subspace identification method SSI-COV, whose implementation in MatLab was previously developed at FEUP (Magalhães 2004).

4.3 *Covariance driven Stochastic Subspace Identification*

The covariance driven Stochastic Subspace Identification method (SSI-COV) performs the identification of a stochastic state space model based on correlations. From the identified model, the estimation of natural frequencies, modal damping ratios and mode shapes is straightforward. All the details of this identification algorithm are explained in (Peeters 2000).

In presence of several setups, the method is normally applied independently for each setup and then final estimates of the modal parameters are obtained using the same procedure that was described in the context of the PolyMax method. In this application this procedure was followed and the obtained results are summarized in Figure 13 (SSI-COV multiple) by the average values and intervals of variation for the damping estimates based on the observed standard deviations.

As an alternative, all the modal parameters can be identified in a single analysis. To perform a single analysis, the correlation matrices of each setup with dimension l by r (l – number of measured degrees of freedom, r – number of reference degrees of freedom) are stacked in a large matrix with dimensions $n.l$ by r (n – number of setups). Following the standard procedure of the SSI-COV method, the correlation matrices evaluated at different time instants are organized in a Toeplitz matrix with dimensions $n.l.i$ by $r.i$ (i – one half of the maximum time lag of the correlation functions). Then, the matrices of the state space model are obtained from the outputs of the singular value decomposition of the Toeplitz matrix. This mathematical operation is the critical step of the method, because the application to a very large matrix can lead to memory problems.

This single analysis produces mode shapes with $n.l$ components, which include n estimates for the reference degrees of freedom and one estimate for the degrees of freedom measured by the moving sensors. The modal ordinates of this last group of degrees of freedom are scaled using the redundant estimates of the references (Parloo 2003).

Figure 11 shows the stabilization diagram provided by the analysis of all setups, that involved the singular value decomposition of a matrix with dimensions 10400x400 (n = 13, l = 8, r = 4 and i = 100). All the natural frequencies of the viaduct can be identified from this very “clean” diagram.

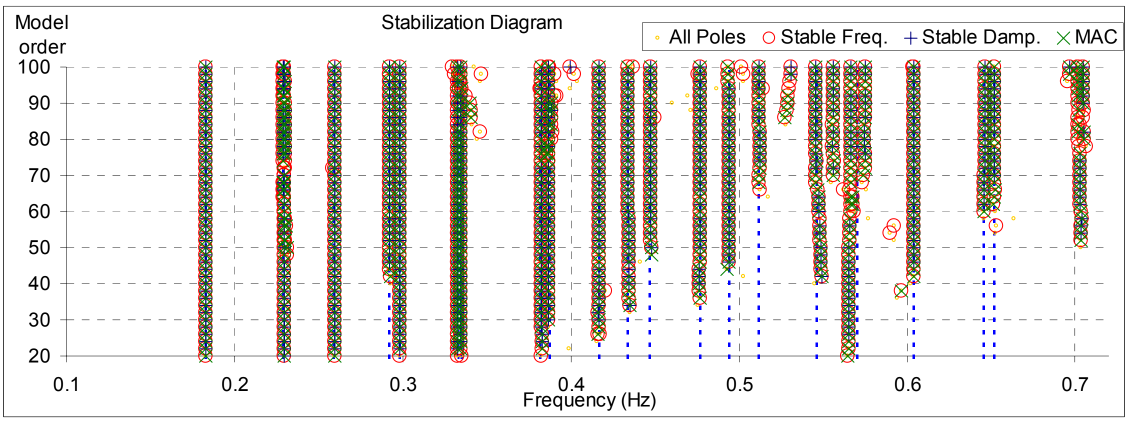

Figure 11. Stabilization diagram of the single analysis – SSI-COV

Figure12 represents the mode shape that was not completely characterized using the multiple analyses, showing that it was well identified with this alternative procedure.

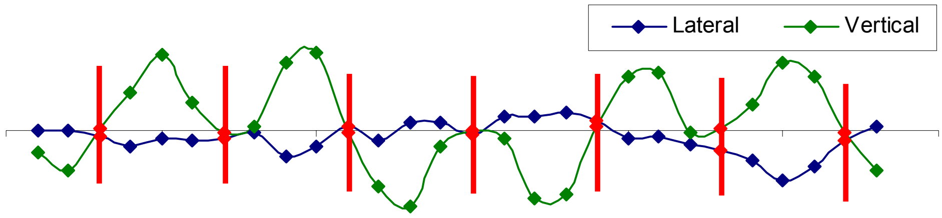

Figure 12. 4th vertical mode shape (f = 0.386Hz), not well identified with the multiple analyses

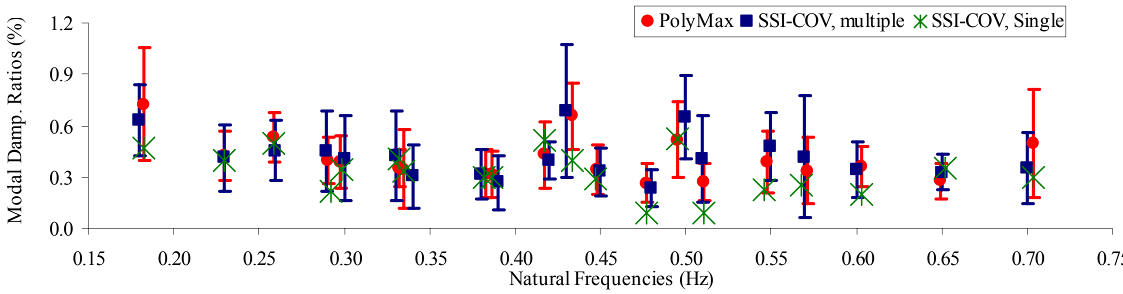

Figure 13. Comparison of the results provided by the applied methods

Figure 13 resumes the results of all the applied techniques, evidencing that the results achieved present a good coherence.

5 OUTPUT-ONLY IDENTIFICATION FROM THE FREE VIBRATION TESTS

The data recorded during the free vibration tests was processed by application of band-pass filters and by the SSI-COV method, adapted to deal with free decays. Good agreement was found between the results obtained from the different data sets and identification methods. Identified natural frequencies were almost coincident with the corresponding estimates achieved with the ambient vibration test, although some frequencies haven't been sufficiently excited to allow accurate identification.

It's worth mentioning that the results provided by the SSI-COV method applied to the data colleted by the monitoring system installed by SITES are particularly interesting. In effect, this identification algorithm confirmed the modal damping ratios identified by the application of band-pass filters and made possible the identification of the damping coefficients of a higher number of modes.

Figure 14 compares the estimates given by the SSI-COV method applied independently to the data of the 13 setups of the ambient vibration test (represented by the average value and by an interval of variation equal to twice the observed standard deviations) with the estimates provided by the SSI-COV method applied to the free decays produced by the sudden cut of the tensioned cable. Imp1 is associated with the force of 600kN and Imp2 with 1000kN and so gives damping estimates for higher amplitudes. Of course the application of the impulses didn't allow the identification of all the modes identified with the ambient vibration test, because this load only excited the vertical modes with non-zero modal ordinates in the point where the impulse was applied. The comparison shows very small differences related with the modes between 0.2 and 0.3 Hz. More significant differences are observed in the third mode. These can be justified by the dependence of the estimates of the modal damping ratio of this mode with the amplitude of vibrations, a coherent increase of the damping ratio with the amplitudes of vibration of the used time segments is observed. For the other modes the results present a reasonable consistency, except for the last mode using Imp2. The stabilization diagram used to obtain this last estimate did not present a clear alignment of stable poles for this mode, and so a reliable result was not expected.

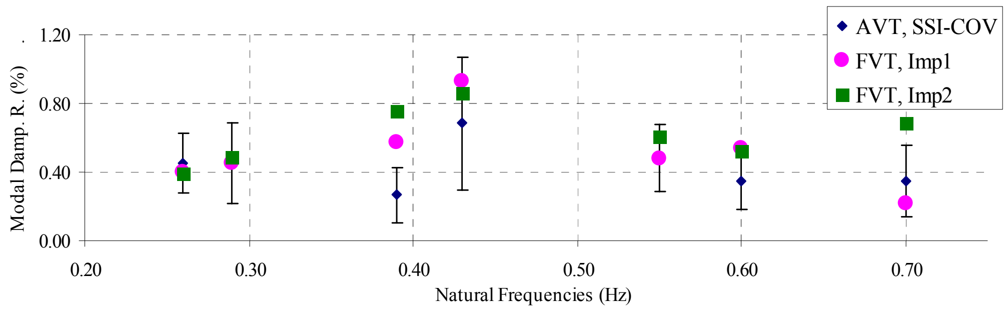

Figure 14. Modal damping ratios obtained from the Ambient Vibration Test (AVT) and from the Free Vibration tests (FVT). Imp1 – 600kNmass; Imp2 – 1000kN mass.

6 CORRELATION WITH THE NUMERICAL MODEL

The application of different stochastic methods of modal identification allowed to obtain very consistent estimates of a significant number of natural frequencies and modes of vibration in the frequency range 0-1Hz. Figures 15 and 16 show 20 modes identified in that range (10 vertical and 10 transversal bending modes), using the SSI-COV method (single analysis), as well as the corresponding values calculated at the design stage (Flamand and Grillaud 2005).

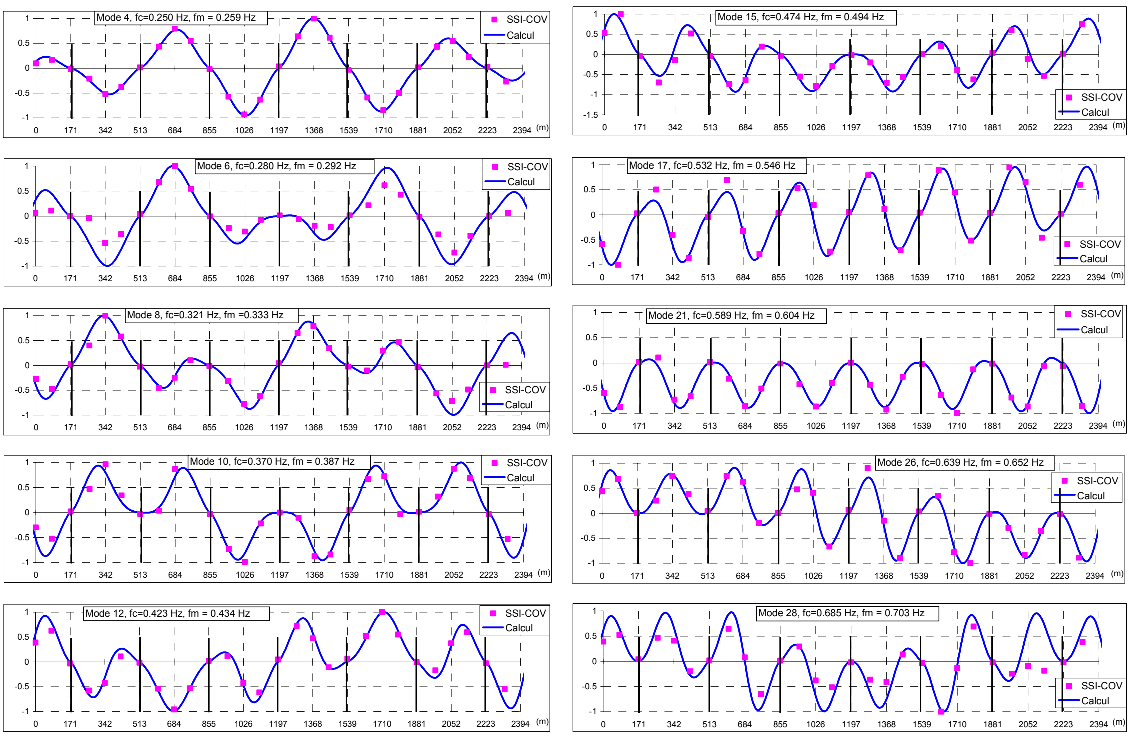

Figure 15. Vertical bending modes in the range 0.2-0.7Hz: identified *vs* calculated modal components

These figures show an excellent agreement between identified and calculated vertical modes, the calculated frequencies being very slightly higher than the identified ones. It is also possible to observe an excellent agreement between the first transversal identified and calculated bending modes, though not so good for modes of higher order. This fact stems probably from less perfect modelling of the piers-foundation interaction, which plays a more significant role in the shortest piers.

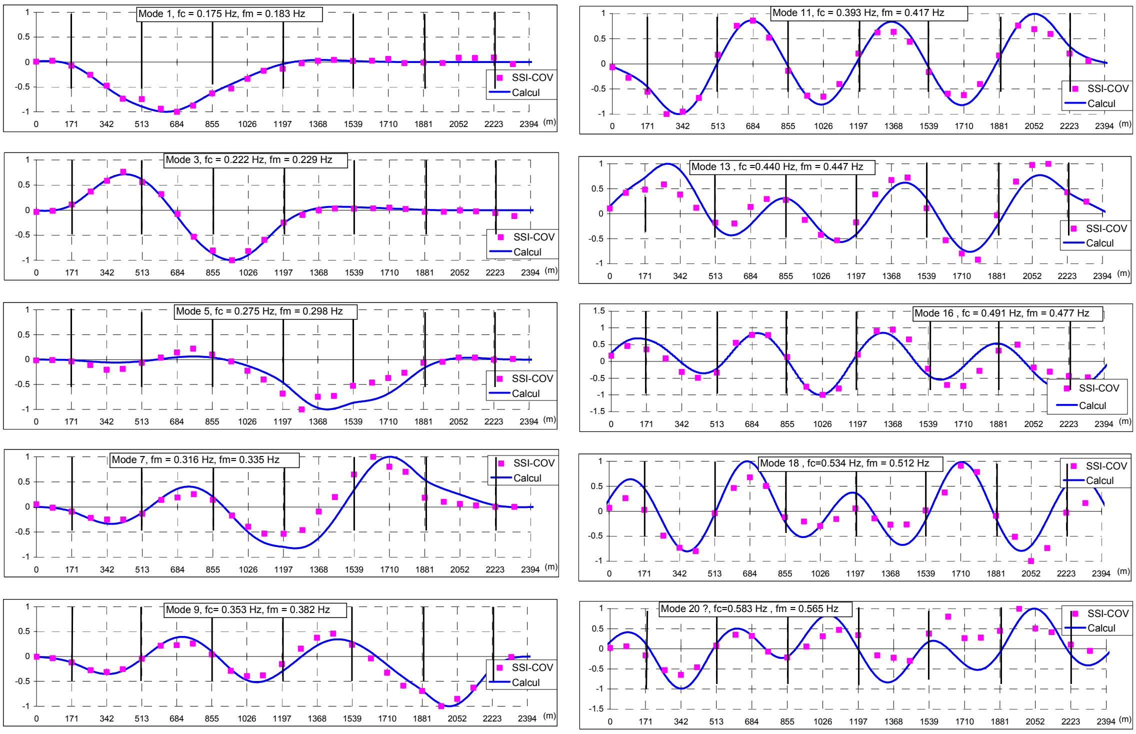

Figure 16. Transversal bending modes in the range 0.1-0.6Hz: identified *vs* calculated modal components

7 CONCLUSIONS

The outstanding characteristics of the Millau Viaduct have motivated the interest for the performance of dynamic tests at the commissioning stage, with the purpose of validating the numerical design studies.

The application of modern stochastic modal identification techniques (EFDD, Polymax and SSI-COV methods) enabled the estimation of about twenty closely spaced modes of vibration in the frequency range 0.1-0.7Hz, exhibiting an excellent correlation with the modal properties previously calculated using the design finite element modelling. This fact shows, on one hand, the excellent numerical structural characterisation and, on the other hand, the excellent performance and high accuracy of the output-only modal identification techniques employed, considering in particular the extremely low levels of vibration of the structure tested in a quiet day without significant wind.

The results obtained from application of Polymax and SSI-COV methods to the ambient vibration data led to very similar estimates not only of natural frequencies and modal shapes, but even of modal damping ratios, whose estimates (average values in the range of 0.2%-0.7%) were also rather consistent with regard to those obtained from the free vibration tests. However, the implementation of a global analysis of the several measurement setups, avoiding the individual analysis of a significant number of stabilization diagrams made the identification procedure much more simple and efficient, allowing even the identification of some modal components that could not be easily extracted from some setups in the classical individual analysis.

8 ACKNOWLEDGEMENTS

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